Math 308L - Autumn 2017
Midterm 1
October 18, 2017

Name: ______________________________
Student ID Number: ______________________________

Question	Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	
Total:	60	

- There are 5 problems on this exam. Be sure you have all 5 problems on your exam.
- The final answer must be left in exact form. Box your final answer.
- You are allowed the TI-30XIIS calculator. It is possible to complete the exam without a calculator.
- You are allowed a single sheet of 2-sided handwritten self-written notes.
- You must show your work to receive full credit. A correct answer with no supporting work will receive a zero.
- Use the backsides if you need extra space. Make a note of this if you do.
- Do not cheat. This exam should represent your own work. If you are caught cheating, I will report you to the Community Standards and Student Conduct office.

Conventions:

- I will often denote the zero vector by 0.
- When I define a variable, it is defined for that whole question. The A defined in Question 2 is the same for each part.
- I often use x to denote the vector $(x_1, x_2, \ldots, x_n)$. It should be clear from context.
- Sometimes I write vectors as a row and sometimes as a column. The following are the same to me.

$$(1,2,3) \qquad \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

- I write the evaluation of linear transforms in a few ways. The following are the same to me.

$$T(1,2,3) \quad T((1,2,3)) \quad T\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right)$$

1. Consider the linear system of equations with the following augmented matrix:

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 3 & 1 \\ 0 & 1 & -1 & 2 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

(a) (8 points) What is the general solution to this system of equations?

(b) (2 points) Write down 2 particular solutions to this system of equations.

(c) (2 points) What is the dimension of the solution space?

2. Let A be the matrix

$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ to be the linear transform defined by $T(x) = Ax$.

(a) (3 points) What is the general solution to $Ax = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$?

(b) (3 points) Give 2 nontrivial solutions to $Ax = 0$.

(c) (3 points) Is T is one-to-one? If not, give a nontrivial solution to $T(x) = 0$.

(d) (3 points) Is T onto? If not, give a vector, b, such that $T(x) = b$ has no solution.

3. Give an example of each of the following. If it is not possible, write "NOT POSSIBLE".
   (a) (2 points) Give an example of a linear system of equations with more equations than variables and exactly one solution.

   (b) (2 points) Give an example of a linear system of equations with more variables than equations and exactly one solution.

   (c) (2 points) Give an example of a set of 4 distinct vectors in $\mathbb{R}^3$ that do not span $\mathbb{R}^3$.

   (d) (2 points) Give an example of a linearly dependent set, S, of 3 vectors such that if we choose any pair of distinct vectors u, v in S, we have that u is not a scale multiple of v.

   (e) (2 points) Give an example of a linear transformation from $\mathbb{R}^3 \to \mathbb{R}^2$ that is onto.

   (f) (2 points) Give an example of a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that $T(1, 0) = (2, 3)$ and $T(2, 0) = (3, 4)$.

4. Let $S = \{u_1, u_2, u_3\}$ be a set of vectors in $\mathbb{R}^4$, where

$$u_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix} \quad u_2 = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad u_3 = \begin{bmatrix} 4 \\ 1 \\ 3 \\ 5 \end{bmatrix}.$$

(a) (6 points) It turns out S is not linearly independent. Show this by writing u_3 as a linear combination of u_1 and u_2.

(b) (3 points) Write u_1 as a linear combination of u_2 and u_3.

(c) (3 points) It should be clear that S does not span $\mathbb{R}^4$. How many additional vectors are required to span $\mathbb{R}^4$? Be sure to briefly justify your answer.

5. Let

$$u_1 = (1, 0, 0), u_2 = (1, 1, 0), u_3 = (1, 1, 1).$$

   (a) (3 points) Express $(0, 1, 0)$ as a linear combination of u_1, u_2, u_3.

   (b) (3 points) Express $(0, 0, 1)$ as a linear combination of u_1, u_2, u_3.

   (c) (3 points) Suppose $T : \mathbb{R}^3 \to \mathbb{R}^2$ is a linear transformation such that $T(u_1) = (1, 0), T(u_2) = (0, 1)$, and $T(u_3) = (1, 1)$. There exists a matrix A such that $T(x) = Ax$. What is A? (Hint: Use the first 2 parts and the fact that T is linear.)

   (d) (3 points) Is T one-to-one? Is it onto? Briefly explain why. (Hint: This part does not require the correct solution to the 3rd part.)